THEORETICAL INVESTIGATION OF THE BURNING OF A MULTIFRACTIONAL COKE DUST FROM AN ASH-RICH FUEL OF LOW DENSITY

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We present the basic results from a theoretical investigation into the burning of a multifractional coke dust from an ash-rich fuel with a low initial density. In addition, we present the formulas to calculate the burnout factor.

It was demonstrated experimentally in [1, 2] that in burning the coke of a fuel of low initial density ($\rho_0 <$ < 0.2 g/cm^3) that is rich in ash, the surface of the active pores within the particle is proportional to the instantaneous density of the fuel. If the particle under consideration is spherical in shape and if its surface is equally accessible to a flow of oxygen, then without consideration of the reduction of the carbon dioxide by the carbon and the volumetric complete combustion of the carbon monoxide (this is quite permissible when burning particles of small dimensions [3]) the combustion process is described by the following system of dimensionless equations [1]:

$$\left.\begin{aligned} &\frac{\partial \eta}{\partial \varphi_0} - b^2 y_0 (1-\eta) = 0, \\ &\frac{\partial \eta}{\partial \varphi_0} - \frac{\partial^2 y_0}{\partial \psi^2} - \frac{2}{\psi}\frac{\partial y_0}{\partial \psi} = 0. \end{aligned}\right\} \qquad (1)$$

For the initial and boundary conditions

$$\eta = 0, \quad y_0 = y(\psi) = 1 \text{ when } \varphi_0 = 0, \quad \left.\frac{\partial \eta}{\partial \psi}\right|_{\psi=0} = 0,$$

from the system of equations (1) we derive the equation

$$\frac{d^2\eta}{d\psi^2} + \frac{1}{1-\eta}\left(\frac{d\eta}{d\psi}\right)^2 + \frac{2}{\psi}\frac{d\eta}{d\psi} + b^2\eta(1-\eta) = 0. \qquad (2)$$

The solution of (2) for the corresponding boundary conditions makes it possible to establish the relationship between the mean particle burnout factor $\bar{\eta} = 3\int_0^1 \eta\psi^2 d\psi$ and the dimensionless time φ_0, as well as its relationship to the decisive parameter b of the combustion process. The results from a numerical integration of (2) for the boundary condition $y_0(\varphi_0)|_{\psi=1} = 1$ (independent of φ_0) to values of $\varphi_0 = 0.19$ are presented in [1] in the form of curves. We see that $\varphi_0 = 0.19$ when b = 2, which corresponds approximately to $\eta =$ $= 0.5$.

To solve the problem within wider limits of φ_0 and b, we integrated Eq. (2), with consideration of external oxygen diffusion, on a Ural-4 computer. In this case, the condition at the outside boundary of the particle is

$$\varphi_0 = -\left.\frac{\ln(1-\eta)}{b^2}\right|_{\psi=1} + \frac{\bar{\eta}}{3\varepsilon}. \qquad (3)$$

The problem was solved for the interval b = 0.5–100 and $\varepsilon = 1.5{-}\infty$ to values of φ_0, corresponding to burning rate $\bar{\eta} = 0.9999$.

The average burnout factor $\bar{\eta}$ for the particle as a function of φ_0 and b when $\varepsilon = 3.5$, derived on the basis of integrating Eq. (2), is shown in Fig. 1. We denote

$$\bar{\eta} = F(\varepsilon, b, \varphi_0). \qquad (4)$$

When b = 0, the burning rate for the particle is defined exclusively by the kinetics of the process, i.e., the burning process takes place within an intrakinetic regime (combustion within the intrakinetic region). In this case the burnout factor

$$\bar{\eta} = 1 - \exp(-b^2\varphi_0). \qquad (5)$$

Formula (5) was derived from the integration of the first equation in system (1) on the assumption that the burning takes place in a medium with a constant oxygen concentration ($y_0 = 1$).

Curve I (Fig. 1) has been plotted on the basis of formula (5) for b = 1. Since the diffusion resistance in this case does not retard the burning process, this curve is situated somewhat higher than the corresponding curve in which diffusion resistance has been taken into consideration. Beginning from b = 1.5, the effect of oxygen diffusion does not exceed 5% [1]. When $b = \infty$, the burning rate for the particle is a function exclusively of the intensity of the internal and external oxygen diffusion. The burning process in this case takes place in the diffusion regime (in the diffusion region). To derive an expression for the burnout factor of the particle in the case of combustion in the diffusion region, we proceed from the following formulas for the specific surfaces of the burning rates [4]:

$$K_S = \frac{\beta c_0}{\dfrac{\Delta}{\mathrm{Nu}_g D} + \dfrac{\Delta}{2D_i}\left(\dfrac{\Delta}{\delta_0} - 1\right)} \qquad (6)$$

and

$$K_S = -\frac{1}{2}\rho_0\left(\frac{\delta_0}{\Delta}\right)^2\frac{d\delta_0}{d\tau}. \qquad (7)$$

From (6) and (7) (in the burning of a particle in a medium with a constant oxygen concentration) we obtain

$$\varphi_0 = \frac{1}{3}\left(\frac{1}{\varepsilon} - 1\right)\bar{\eta} + \frac{1}{2}\left[1 - \left(\sqrt[3]{1-\bar{\eta}}\right)^2\right]. \qquad (8)$$

Beginning from b = 15, the effect of the kinetic factors is small and the burning process may be treated, practically speaking, as taking place within the diffusion region. This can be seen from Fig. 1.

Let the dust system contain particles ranging in diameter from 0 to Δ_{max}. Here, proceeding from the specified accuracy, we determine the diameter of the largest particle in the multifractional dust system from the condition $R(\Delta_{max})$ = const. Here $R(\Delta)$ expresses the law of mass particle distribution on the basis of the dimensions of the particles in the multifractional system. It is thus obvious that the individual particles in the system also exhibit various values for the decisive parameter in the combustion process, i.e., $b = \Delta(Ak\rho_0/4D_i)^{1/2}$, which varies from 0 to b_{max}.

The initial density ρ_0 of the fuel particles depends on the amount of volatile substance and the quantity of ash in the fuel. In grinding fuels containing rock inclusions (e.g., carbonates in shales), the value of ρ_0 must be determined on the basis of that fraction of the ash which is not separated from the fuel during the grinding process. The value of $R(\Delta)$ must also be determined in the light of this phenomenon.

Let us assume that all of the particles in the multifractional dust system burn at a given instant of time under conditions of equal oxygen concentrations and equal temperatures. We also assume that for all particles $Ak\rho_0/D_i$ = const and ε = const. This makes it possible to express the burning rate for this particle in terms of the burnout factor for the largest particle in the system [5].

On the basis of (4)

$$\left.\begin{aligned}\varphi_{0\,max} &= F^{-1}(b_{max}, \bar{\eta}_{max}),\\ \varphi_0 &= F^{-1}(b, \eta),\end{aligned}\right\} \qquad (9)$$

where $\varphi_{0\,max}$ and φ_0, respectively denotes the dimensionless time for particles of diameters Δ_{max} and Δ. Since

$$\varphi_{0\,max} = \frac{4D_i\beta c_0}{\rho_0}\,\frac{\tau}{\Delta^2_{max}} \text{ and } \varphi_0 = \frac{4D_i\beta c_0}{\rho_0}\,\frac{\tau}{\Delta^2},$$

then $\varphi_0 = \varphi_{0\,max}/\Delta_0^2 = \varphi_{0\,max}/(b/b_{max})^2$ and on the basis of formula (9)

$$F^{-1}(b_{max}, \bar{\eta}_{max}) = \left(\frac{b}{b_{max}}\right)^2 F^{-1}(b, \bar{\eta}). \qquad (10)$$

Bearing in mind that $b = \Delta_0 b_{max}$,

$$F^{-1}(b_{max}, \bar{\eta}_{max}) = \left(\frac{b}{b_{max}}\right)^2 F^{-1}(\Delta_0 b_{max}, \bar{\eta}). \qquad (11)$$

For fixed values of b_{max} the last equation makes it possible to establish the function $\bar{\eta} = \eta(\bar{\eta}_{max}, \Delta_0)$.

The value of b_{max} defines the burning regime for the largest particle in the multifractional dust system. For large b_{max} the effect of the kinetic factors is small and Eq. (11) with consideration of (8) assumes the form

$$\frac{1}{3}\left(\frac{1}{\varepsilon}-1\right)\bar{\eta}_{max} + \frac{1}{2}\left[1-\left(\sqrt[3]{1-\bar{\eta}_{max}}\right)^2\right] = \Delta_0^2 F^{-1}(b, \bar{\eta}). \qquad (12)$$

In the limit regime, when all of the particles in the system burn within the diffusion region, Eq. (12) is additionally simplified:

$$\frac{1}{3}\left(\frac{1}{\varepsilon}-1\right)\bar{\eta}_{max} + \frac{1}{2}\left[1-\left(\sqrt[3]{1-\bar{\eta}_{max}}\right)^2\right] = \Delta_0^2\left\{\frac{1}{3}\left(\frac{1}{\varepsilon}-1\right)\bar{\eta} + \frac{1}{2}\left[1-\left(\sqrt[3]{1-\bar{\eta}}\right)^2\right]\right\}. \qquad (13)$$

Figure 2 shows the burnout factor for a particle in the system as a function of its relative diameter for various $\bar{\eta}_{max}$ and for three values of b_{max}.

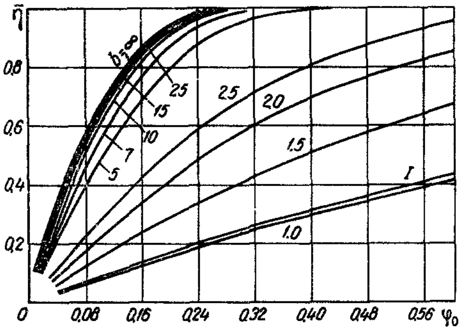

Fig. 1. Burnout factor $\bar{\eta}$ versus dimensionless time φ_0 at various values of b (ε = 3.5). I) $\bar{\eta}$ – – exp φ_0.

The curves for b_{max} = 5 are plotted according to Eq. (11). Since the combustion takes place within the intrakinetic region when $b \le 1.5$, the relative particle diameter which defines the boundary between the intrakinetic and intermediate regions, is equal to $\Delta_{0\,kin}$ = = 0.3. Particles with relative dimensions greater than 0.3 burn in the intermediate regions. In the interval of relative dimensions $\Delta_0 = 0 - \Delta_{0\,kin}$ the relationship between $\bar{\eta}$ and Δ_0 is expressed by means of horizontal straight lines, since when b = 0 the burning rate is independent of the external particle dimensions (Formula (5)).

The curves for $b_{max} = \infty$ (dashed lines) are plotted according to Eq. (13). Each value of $\bar{\eta}_{max}$ corresponds to a specific limit diameter $\Delta_{0\,lim}$ below which all of the particles are completely burned. On the basis of Eq. (13)

$$\Delta_{0\,lim} = \left[\frac{\frac{1}{3}\left(\frac{1}{\varepsilon}-1\right)\bar{\eta}_{max} + \frac{1}{2}\left[1-\left(\sqrt[3]{1-\bar{\eta}_{max}}\right)^2\right]}{\frac{1}{6}+\frac{1}{3\varepsilon}}\right]^{1/2} \qquad (14)$$

The curves for b_{max} = 50 are plotted according to Eq. (12). Obviously, $\Delta_{0\,kin}$ = 0.03. Since $b_{max} > 15$, in the interval of the relative diameters Δ_0 = 0.3–1.0 the particles burn in the diffusion region. Therefore, beginning from Δ_0 = 0.3, the curves for b_{max} = 50 coincide with the curves of pure diffusion combustion.

For a mathematical description of the quantitative relationships governing the mass distribution of the

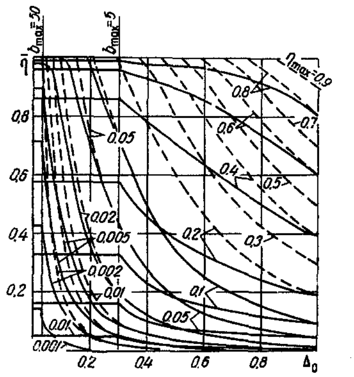

Fig. 2. Burnout factor of particles versus their relative sizes at various values of $\bar{\eta}_{max}$ (ε = 3.5).

disperse material on the basis of the particle dimensions we frequently employ the Rozin-Rammler law. However, the various properties of the ground materials are most completely described by the normal-logarithmic law

$$R(\Delta)=\frac{1}{2}\left\{1-\Phi\left[\ln\left(\frac{\Delta}{\Delta_s}\right)^{m_0}\right]\right\}. \qquad (15)$$

Here $\Phi(x)$ denotes the Gaussian probability integral

$$\Phi(x)=\frac{2}{\sqrt{2\pi}}\int_0^x \exp\left(-\frac{t^2}{2}\right)dt.$$

In analyzing the burning process of a multifractional fuel-dust system, it is best to use the diameter of the largest particle in the place of the mass median diameter of the system which figures in formula (15)

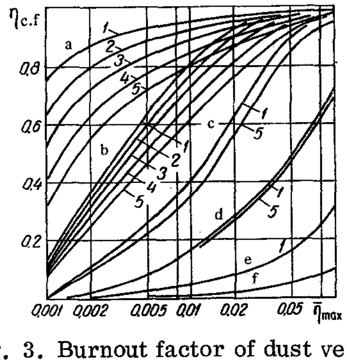

Fig. 3. Burnout factor of dust versus $\bar{\eta}_{max}$ at various values of m_0 and b_{max} (ε = 3.5, $R(\Delta_{max})$ = 0.001): a) b_{max} = = ∞; [1) m_0 = 0.7; 2) 0.8; 3) 0.9; 4) 1.0; 5) 1.1]; b) b_{max} = 50 [1–5) the same]; c) 25; d) 10; e) 5; f) 0.

and to express the normal-logarithmic laws in terms of the relative particle dimensions. For this we give formula (15) the form

$$R(\Delta_0)=\frac{1}{2}\{1-\Phi[\ln(\lambda\Delta_0)^{m_0}]\}. \qquad (16)$$

The coefficient λ is a function of the chosen value for $R(\Delta_{max})$.

Using the law expressed by formula (16) for the distribution of the particles in the multifractional system, we find the burnout factor for the dust in the form

$$\eta_{c.f}=\int_1^0 \eta(\bar{\eta}_{max},\ b_{max},\ \Delta_0)\frac{dR(\Delta_0)}{d\Delta_0}d\Delta_0=$$

$$=\frac{1}{2}\int_0^1 \eta(\bar{\eta}_{max},\ b_{max},\ \Delta_0)\frac{d\Phi[\ln(\lambda\Delta_0)^{m_0}]}{d\Delta_0}d\Delta_0. \qquad (17)$$

Integration of expression (17) yields the function $\eta_{c.f}=\eta(m_0, b_{max}, \bar{\eta}_{max})$. Some results of the integration of (18) are shown in Fig. 3. Analysis of these functions permits us to establish certain general quantitative relationships.

With approach of the multifractional dust system in terms of its granular characteristics to a monofrac-

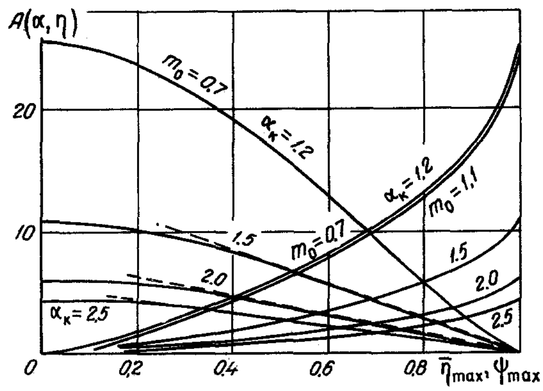

Fig. 4. $A(\alpha,\eta)$ versus $\bar{\eta}_{max}$ (ε = 3.5), b_{max} = 25.

tional composition, which corresponds to an increase in the index of dust homogeneity, the curves $\eta_{c.f}$ = = $\eta(\bar{\eta}_{max})$ approach the maximum value of $\eta_{c.f}$ more slowly. The increase in the parameter b_{max} proceeds in the same direction. It is obvious that the smaller the value of b_{max}, the closer the burning regime for the multifractional dust to the burning regime in the intrakinetic region (if b_{max} = 0, then $\eta_{c.f}=\bar{\eta}_{max}$). When $b_{max} \le 10$, the effect of the multifractional dust on the nature of the curves $\eta_{c.f}=\eta(\bar{\eta}_{max})$ is insignificant.

The partial volume of the oxygen at the beginning of the coke-burning zone r_{0O} on subsequent burning of the volatile substances and the coke depends on the overall coefficient of excess air, as well as on the quantity and elementary composition of the volatile substances. In the assumption that the theoretical quantities of air needed for the burning of the coke and the volatile substances are equal, r_{0O} is expressed by the formula

$$r_{0_O}=\frac{1-\dfrac{\beta_v}{\alpha_O(1-\beta_v)+\beta_v}}{1+\varkappa}r_{O_2}, \qquad (18)$$

since the coefficient for the increase in the gas volume $\varkappa = (\beta_v V_c/\alpha V^0) - 1$ and the coefficient for the excess air in the coke-burning zone $\alpha_O = (\alpha - \beta_v)/(1 - \beta_v)$.

Considering that the instantaneous partial volume of the oxygen in the coke-burning zone is related to the burnout factor for the multifraction dust by the relationship $r_{0O} = r_{0O}(1 - \eta_{c.f}/\alpha_O)$, the time for the process in the general case is given by

$$\tau = \frac{\rho_0 \Delta_{max}^2 (1+\varkappa)\left(\alpha_O + \frac{\beta_v}{1-\beta_v}\right)}{4\beta c_0 m D} \times \int_0^{\bar\eta_{max}} \frac{d\bar\eta_{max}}{(\alpha_O - \eta_{c.f})\left(\frac{d\bar\eta_{max}}{d\varphi_0}\right)}. \quad (19)$$

We denote

$$\gamma(\Delta_{max}) = \frac{\rho_0 \Delta_{max}^2}{4\beta c_0 D}$$

and

$$A(\alpha, \eta) = \frac{(1+\varkappa)\left(\alpha_O + \frac{\beta_v}{1-\beta_v}\right)}{m} \times \int_0^{\bar\eta_{max}} \frac{d\bar\eta_{max}}{(\alpha_O - \eta_{c.f})\left(\frac{d\bar\eta_{max}}{d\varphi_0}\right)},$$

so that

$$\tau = \gamma(\Delta_{max}) A(\alpha, \eta). \quad (20)$$

Depending on the burning regime, the function $A(\alpha, \eta)$ may have various forms. Thus, for example, in the burning of the largest particle in the diffusion region

$$A(\alpha, \eta) = \frac{(1+\varkappa)\left(\alpha_O + \frac{\beta_v}{1-\beta_v}\right)}{3m} \times \int_0^{\bar\eta_{max}} \frac{\frac{1}{\varepsilon} + \frac{1}{\sqrt[3]{1-\bar\eta_{max}}} - 1}{\alpha_O - \eta_{c.f}} d\bar\eta_{max}. \quad (21)$$

Figure 4 shows $A(\alpha,\eta)$ as a function of the burnout factor $\bar\eta_{max}$ of the largest particle (or of the relative diameter ψ_{max} of the unburned core), plotted from the results of the integration of expression (21) when $b_{max} = 25$. It develops that $A(\alpha,\eta)$ is a strong function of the excess-air coefficient. In first approximation $A(\alpha,\eta) \sim \alpha_O^{-2}$. The index of dust homogeneity has a relatively weak effect on $A(\alpha,\eta)$, particularly in the case of high excess-air coefficients.

For high values of $b_{max} > 25$, beginning from $\eta_{max} \geq 0.25$ and $\alpha_O > 1.5$, the relationship between $A(\alpha,\eta)$ and ψ_{max} is approximated by the rectilinear relationship (Fig. 4)

$$A(\alpha, \eta) = 28\,\alpha_O^{-\frac{9}{5}}(1 - \psi_{max}) = 28\,\alpha_O^{-\frac{9}{5}}\left(1 - \sqrt[3]{1-\bar\eta_{max}}\right). \quad (22)$$

The results derived here can be used to analyze the effect of individual parameters (in particular, the excess-air coefficient, temperature, fineness of fuel grinding) on the process of burning ash-rich coke fuel dust, as well as to evaluate the burning rate.

NOTATION

ρ is the density of the fuels in the particle; $\varepsilon = Nu_g/2m$; η and $\bar\eta$ are the local and mean burnout factors for the particle; $\bar\eta_{max}$ is the mean burnout factor of the largest particle in multifractional dust system; $\eta_{c.f}$ is the burnout factor of coked multifractional dust; $m = D_i/D$; τ is the time; β is the stoichiometric coefficient; A is the specific active pore surface in the particle; k is the burning-rate constant; c is the oxygen concentration at a given point; c_0 is the initial concentration of oxygen in the surroundings; y_0 is the relative concentration of oxygen; D and D_i are the coefficients of outside and inside diffusion of oxygen; δ and Δ are the instantaneous and outside particle diameters; δ_0 is the diameter of unburned core in the diffusion region; ψ is the relative instantaneous particle diameter; Δ_{max} is the diameter of largest particle in the system; Δ_0 is the relative particle diameter; ψ_{max} is the relative diameter of the unburned core in the diffusion region; Δ_s is the mass median diameter of polyfractional dust system; $\lambda = \Delta_{max}/\Delta_s$; $\varphi_0 = 4\beta c_0 D_i \tau/\Delta^2 \rho_0$ is the dimensionless time; b is the determining parameter for burning; Nu_g is the diffusion Nusselt number; m_0 is the index of dust uniformity; α is the total coefficient of excess air; β_v is the relative volatile content in the fuel; V^0 is the theoretical amount of air necessary for complete fuel burnout; V_c is the amount of fuel burnout products; r_{0O} is the partial volume of oxygen in the initial region of coke burning; r_{O_2} is the partial volume of oxygen at the beginning of burning.

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